

Calculators and mobile phones are not allowed.

Answer all of the following questions.
Each (sub)question is worth 5 points.

1. Find the limit:

(a) $\lim_{x \rightarrow 0^+} \frac{\cot x}{\ln x}$

(b) $\lim_{x \rightarrow \infty} (x + e^x)^{\frac{2}{x}}$

2. Evaluate the following integrals:

(a) $\int x(\ln x)^2 dx$

(b) $\int \cot^3 x \csc^4 x dx$

(c) $\int \frac{x^2 + x + 2}{x(x^2 + 2x + 2)} dx$

(d) $\int \frac{dx}{\sqrt[3]{x} - \sqrt{x}}$

(e) $\int \frac{e^{2x}}{\sqrt[3]{1 + e^x}} dx$

(f) $\int \frac{x^2}{\sqrt{4 - x^2}} dx$

3. Determine whether the following integral converges. If it converges, find its value.

$$\int_0^{\infty} \frac{x^2}{1 + x^6} dx$$

4. Find the arc length of the curve C given by the parametric equations

$$x = \frac{\pi}{2} + t - \sin t, \quad y = \cos t - \pi, \quad \frac{\pi}{2} \leq t \leq \pi.$$